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## Concentrated Load on Reinforced Concrete Beam

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# CONCENTRATED LOAD ON REINFORCED CONCRETE BEAM

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According to Danish Standard DS 411 the load carrying capacity of a reinforced concrete beam is determined using the extremum principles of the theory of plasticity. The model used is shown in figure 1.

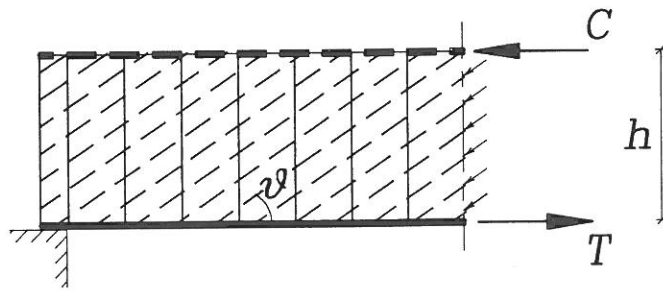


Figure 1

It is a kind of truss model that consists of two horizontal stringers and a zone between the two stringers in which the concrete is in uniaxial compression and the stirrups are in uniaxial tension.

In the concrete the stress components are

$$\sigma_{xx}^c = -\sigma^c \cos^2 \vartheta$$

$$\sigma_{yy}^c = -\sigma^c \sin^2 \vartheta$$

$$\sigma_{xy}^c = -\sigma^c \sin \vartheta \cos \vartheta$$

where  $\sigma^c$  is the uniaxial compressive stress, and  $\vartheta$  is the angle between a horizontal  $x$ -axis and the direction of compression. In the vertical stirrups the stress is  $\sigma^s$  and the components are

$$\sigma_{xx}^s = 0$$

$$\sigma_{yy}^s = \sigma^s$$

$$\sigma_{xy}^s = 0$$

The total stress thus becomes

$$\sigma_{xx} = -\sigma^c \cos^2 \vartheta$$

$$\sigma_{yy} = -\sigma^c \sin^2 \vartheta + \varphi \sigma^s$$

$$\sigma_{xy} = -\sigma^c \sin \vartheta \cos \vartheta$$

where  $\varphi$  is the reinforcement ratio for the stirrups.

Using these assumptions one finds that the stresses  $\sigma^c$  and  $\sigma^s$  are determined from

$$\sigma^c b h \sin \vartheta \cos \nu = Q$$

$$\varphi \sigma^s b h \cot \vartheta = Q$$

where  $b$  is the width of the beam,  $h$  is the distance between the two horizontal stringers and  $Q$  is the shear force.

With these stresses, the stringer forces  $T$  and  $C$  in the horizontal stringers are found

$$T = M/h + \frac{1}{2} Q \cot \nu$$

$$C = M/h - \frac{1}{2} Q \cot \nu$$

where  $M$  is the moment. The variation of  $T$  and  $C$  is shown in figure 2. For more details see [1].

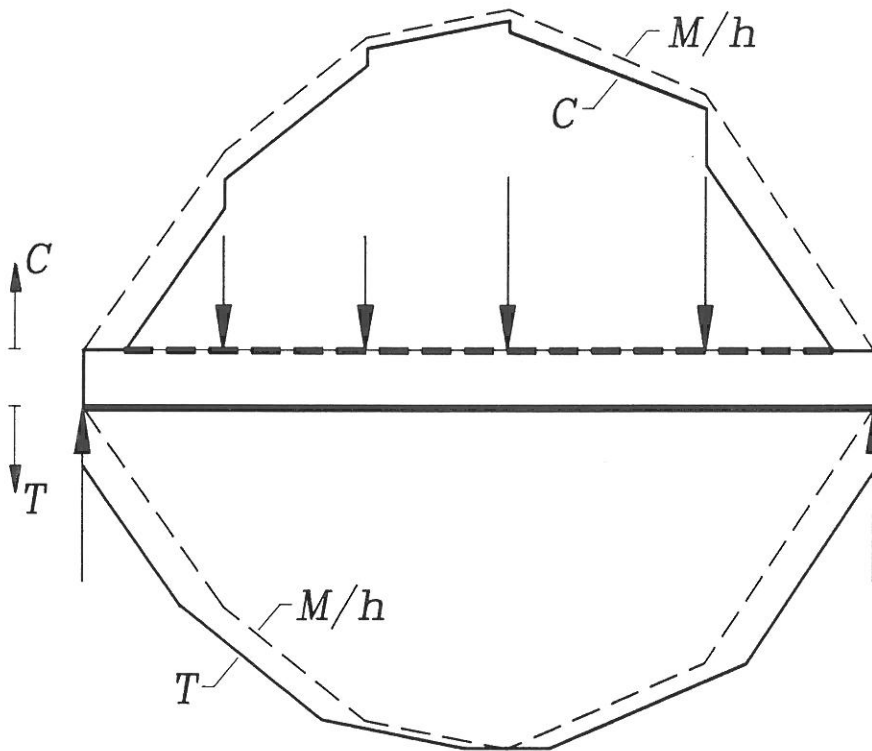


Figure 2

In a beam loaded with concentrated loads, the shear force is constant between any two loads and also the stresses  $\sigma^c$  and  $\sigma^s$  are constant. One question in this connection is: How do the stresses  $\sigma^c$  and  $\sigma^s$  change from values corresponding to one value of the shear force  $Q = Q_1$  to another  $Q = Q_2 = Q_1 - P$ .

An answer to that question is found in assuming the existence of an inclined stringer under the concentrated load as shown in figure 3. The force in this stringer varies

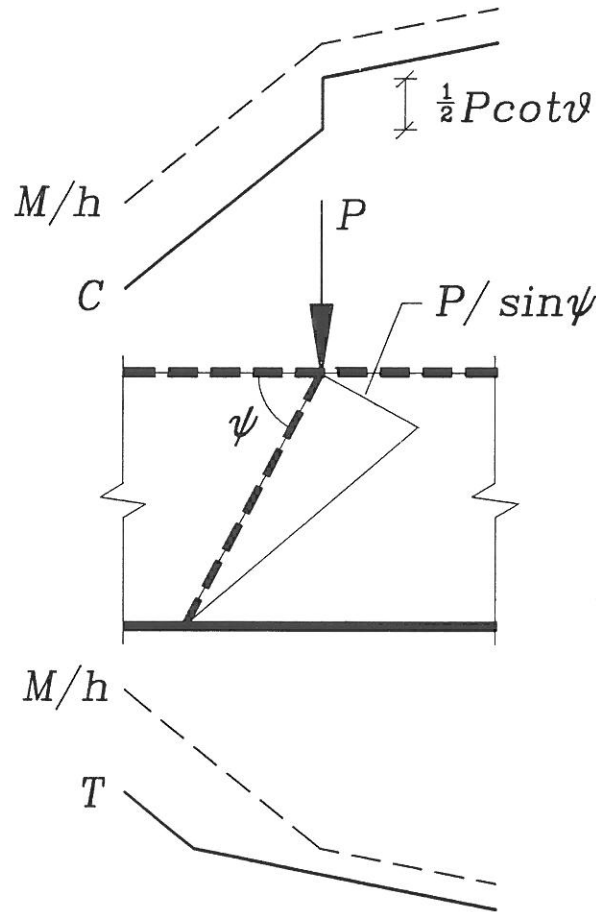


Figure 3

linearly from  $P/\sin \psi$  to 0, where the inclination  $\psi$  is determined from

$$\cot \psi = \frac{1}{2} \cot \vartheta$$

The complete stringer system for a simply supported beam is shown in figure 4.

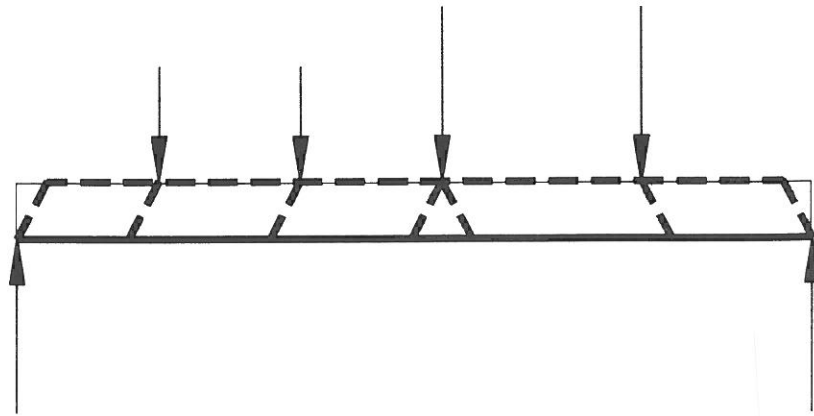


Figure 4

- [1] Nielsen, M.P.: Limit Analysis and Concrete Plasticity, Englewood Cliffs, Prentice-Hall, Inc. 1984.